# Problem Set 1: Energy, power, unit conversions 

Energy and the built environment CRP 570.004/470.004

Total Points: 50 pts
Due Date: $\quad 1 / 28,12 \mathrm{pm}$, in class.

## Instructions for completing problem sets:

1. Mark your answers clearly by circling/boxing them.
2. Clearly state any assumptions (e.g., the price of gasoline, the energy content used for a fuel).
3. Cite all of your data sources.
4. Show all your work so that I can figure out where you went wrong, and award partial credit where applicable.
5. Use scientific notation for writing very small or very large numbers.
6. Use the correct number of significant figures (points will be deducted if you keep to many digits!)
7. Your name must appear at the top of each sheet you turn in (and all must be stapled).
8. Late assignments should be scanned and emailed to me, and will be penalized $10 \%$ for each day late, up to 7 days, after which solutions will be posted and problem sets no longer accepted.
9. You will get the most out of the problem sets if you make an initial effort to work through all of the problems on your own. After attempting to solve the problems on your own, you may then work with other students to discuss different approaches. Even then, you should work out each problem yourself. It is encouraged to compare answers with classmates upon completion of the problem set. Remember that it is a violation of UNM's Academic Honesty to copy answers from anyone.
10. Conversions and energy density. ( 25 pts)

From EIA "Energy sources are commonly measured in different physical units to include barrels of oil, cubic feet of natural gas, tons of coal, and kilowatthours of electricity. In the United States, British thermal units (Btu), a measure of heat energy, is commonly used for comparing different types of energy. In 2013, total U.S. primary energy use was about 97.5 quadrillion ( $10^{15}$, or one thousand trillion) Btu." The energy density of a fuel is the amount of energy it contains on a per unit (mass or volume) basis.

Show your work for each conversion (don't just use an online converter) and keep only significant digits.
a. Wood: $16 \mathrm{MJ} / \mathrm{kg}(3)$

- $\frac{16 M J}{k g} x \frac{1 e 6 J}{1 M J} x \frac{1 \mathrm{kWh}}{3.6 e 6 J}=4.4 \mathrm{kWh} / \mathrm{kg}$
- $\$ 325 /$ cord (http://www.nh.gov/oep/energy/energy-nh/fuel-prices/index.htm) A cord is a unit of volume (3.63m^3) of which 1.85 is actually wood. I will use and average wood density from here (https://cedarstripkayak.wordpress.com/lumber-selection/162-2/) which was $0.63 \mathrm{~kg} / \mathrm{m}^{\wedge} 3$. This isn't a great estimate, since it is treating all the woods as equally available. As well, it doesn't list the moisture content for which the densities are listed.
- $\frac{\$ 325}{1.85 \mathrm{~m}^{3}} \times \frac{\mathrm{m}^{\wedge} 3}{630 \mathrm{~kg}} \times \frac{\mathrm{kg}}{4.4 \mathrm{kWh}}=.063 \$ / \mathrm{kWh}$
b. Coal: $24 \mathrm{MJ} / \mathrm{kg}(3)$
- $\frac{24 M J}{k g} x \frac{1 e 6 J}{1 M J} x \frac{1 \mathrm{kWh}}{3.6 e 6 J}=6.7 \mathrm{kwh} / \mathrm{kg}$
- From http://www.eia.gov/coal/news markets/, I find that for low sulfur coal for Jan 2015 it is pricing at $\$ 63 / 907 \mathrm{~kg}$. High sulfur low energy density coal can run as cheap as $\$ 11 /$ short ton. A short ton is 907 kg .
- $\frac{\$ 63}{907 \mathrm{~kg}} \times \frac{\mathrm{kg}}{6.7 \mathrm{kWh}}=0.01 \$ / \mathrm{kWh}$
c. Gasoline: 112 BTU/gal (you will have to find the density of gasoline) (3)
- $\frac{112000 B T U}{g a l} \times \frac{1 g a l}{3.785 l} \times \frac{l}{0.77 \mathrm{~kg}} \times \frac{\mathrm{kWh}}{3414 B t u}=11 \mathrm{kWh} / \mathrm{kg}$
- $\frac{1.80 \$}{\text { gal }} x \frac{1 \mathrm{gal}}{3.785 l} \times \frac{l}{0.77 \mathrm{~kg}} \times \frac{\mathrm{kg}}{11 \mathrm{kWh}}=0.056 \mathrm{kWh} / \mathrm{kg}$. Estimate using current pump prices in Abq of $\$ 1.80 / \mathrm{gal}$.
d. Crude oil: $6.1 \mathrm{GJ} / \mathrm{bbl}(3)$
- $\frac{6.1 G J}{b b l} \times \frac{1 e 9 J}{1 G J} \times \frac{1 k W h}{3.6 e 6 J} \times \frac{1 b b l}{42 g a l} \times \frac{308 \mathrm{gal}}{1000 \mathrm{~kg}}=12 \mathrm{kwh} / \mathrm{kg}$ (density of crude oil taken as world average, from BP World Energy report.
- $\frac{60 \$}{b b l} x \frac{1 b b l}{42 g a l} \times \frac{308 \mathrm{gal}}{1000 \mathrm{~kg}} \times \frac{\mathrm{kg}}{12 \mathrm{kWh}}=0.037 \$ / \mathrm{kg}$
e. Butter: $717 \mathrm{kcal} / 100 \mathrm{~g}(3)$
- $\frac{717 \mathrm{kcal}}{100 \mathrm{~g}} x \frac{1000 \mathrm{cal}}{1 \mathrm{kcal}} x \frac{4.184 \mathrm{~J}}{1 \mathrm{cal}} x \frac{1 \mathrm{kWh}}{3.6 \mathrm{e} \sigma \mathrm{J}} x \frac{1000 \mathrm{~g}}{1 \mathrm{~kg}}=8.3 \mathrm{kWh} / \mathrm{kg}$
- From (http://www.cheesereporter.com/butteraverages.htm), 2013 ave was $1.5 \$ / \mathrm{lb}, 2014$ was $2.15 \$ / \mathrm{lb}$
- $\frac{2.15 \$}{l b} x \frac{2.2 l \mathrm{~b}}{1 \mathrm{~kg}} x \frac{\mathrm{~kg}}{8.3 \mathrm{kWh}}=0.57 \$ / \mathrm{kg}$ (The price is likely so expensive because it isn't bought in bulk)
$\bullet$
f. Natural gas: 20,160 BTU/lb (3)
- $\frac{20,160 \mathrm{Btu}}{l b} x \frac{2.2 l b}{1 \mathrm{~kg}} \times \frac{\mathrm{kWh}}{3414 \mathrm{Btu}}=13 \mathrm{kWh} / \mathrm{kg}$
- Most likely you will find prices per BTU or per volume (cubic meter/cubic feet). If you find prices per unit of energy or volume then there MUST be some underlying assumption about the density of the gas. Using an average residential price of 12.45 \$/thousand cubic feet in 2013 (EIA,2013). I found that this is usually measured at $0 C$ and 1 atm , so I used an online density calculator to find $0.75 \mathrm{~kg} / \mathrm{m}^{\wedge} 3$ of nat gas.
- $\frac{12.45 \$}{1000 f t^{\wedge} 3} \times \frac{f t^{\wedge} 3}{0.0283 m^{\wedge} 3} \times \frac{m^{\wedge} 3}{0.75 \mathrm{~kg}} \times \frac{\mathrm{kg}}{13 \mathrm{kWh}}=0.0045 \$ / \mathrm{kg}$.
g. The US consumed 12.4 Quads of electricity in 2013.
- How much primary energy was this, in kWh? Assume that conversion of primary energy into end use electricity is typically $32 \%$ efficient. (2)
- $\frac{12.4 e 15 \mathrm{BTU}}{\text { year }} x \frac{1 \mathrm{kWh}}{3414 B T U}=3.63 \mathrm{e} 12 \mathrm{kWh}$ electricity .
- Efficiency can be defined as: $\eta=\frac{\text { What you want out }}{\text { what you put in }}=\frac{\text { electricity }}{\text { primary energy }}$. So, rearranging this equation we can write it as: primary ener gy $=\frac{\text { electricity }}{\eta}=\frac{3.63 \mathrm{e} 12 \mathrm{kWh}}{0.32}=1.1 \mathrm{e} 13 \mathrm{kWh}$
- How many kg of primary energy, in the form of gasoline would be needed to meet this demand? (2)
- From part c) we found that the energy density of $11 \mathrm{kWh} / \mathrm{kg}$ so $1.1 \mathrm{e} 13 \mathrm{kWh} / 11 \mathrm{kWh} / \mathrm{kg}=1.0 \mathrm{e} 12 \mathrm{~kg}$ of gasoline. Current annual gasoline consumption is
- How many kg of primary energy, in the form of wood, would be needed to meet this demand? (2)
- From part a) we found that the energy density of wood is $4.4 \mathrm{kWh} / \mathrm{kg}$ so $1.1 \mathrm{e} 13 \mathrm{kWh} / 4.4 \mathrm{kWh} / \mathrm{kg}=$ 2.5 e 12 kg of wood. This is twice the mass of gasoline!
- Which fuel from the list would be best to use as a transportation fuel? Why? (1)
- There could be many answers here, but in terms of energy density, oil/gasoline makes the most sense because they have high energy density so you'd need less mass - the greater the mass you need to carry in a transport vehicle (especially on land or air) the more energy you need to resist gravity (air) or overcome friction (roads). Natural gas has a high energy density, but a gas isn't great to use because at normal pressures the volume is very high, so you'd need big containers. That's why LNG (liquefied natural gas) is the form used for both transporting it, and as a transportation fuel. Another important point is that the form of oil or gasoline is appropriate for internal combustion engines or turbines,
whereas coal in its solid form is useful for much more than a steam engine which is much less efficient (a steam turbine is efficient, but then you'd also need to be carrying your boiler where you are combusting your coal to heat water and transform it into steam.


## 2. Electricity generation ( 15 pts)

a. In 2013416 MToe of primary energy of coal was used to generate electricity.

- Using the same average efficiency of $32 \%$, how many TWh of electricity was this? (Use the conversion factor from IEA) (3)
- $\frac{416 e 6 \text { Toe }}{y r} \times \frac{41.9 G J}{1 T o e} x \frac{1 e 9 J}{1 G J} \times \frac{1 \mathrm{kWh}}{3.6 e 6 J} \times \frac{1 T W h}{1 e 9 k W h}=4840 \mathrm{TWh} / \mathrm{yr}$ primary $\times 0.32=1550 \mathrm{TWh} / \mathrm{yr}$ electricity
- If total electricity generation was 12.5 Quads, what percent was generated from coal? (3)
- $\frac{12.5 \text { Quads }}{y r} x \frac{293 T W h}{1 \text { Quad }}=3662.5$ TWh (keep sig digs), 1550 TWh/3662.5TWh $=0.423$ or $42.3 \%$ from coal
b. Use the value above of electrical energy generated in a year from coal to calculate the average annual power of the coal generating stations (remember that Power = Energy/time) if you assume that they operated continuously. (3)
- Power = Energy/Time. I didn't specify the units for power, but I will use GW
- $\frac{1550 T W h}{Y r} x \frac{1000 G W h}{1 T W h} x \frac{1 y r}{365 * 24 h}=177 \mathrm{GW}$ average power
c. On the EIA website, find how many coal power plants were operating in 2012. 557 Now, using the average power you calculated above, estimate the average power per coal plant. (2)
- I find there were 557 coal plants, so each coal plant had an average power of $177 \mathrm{GW} / 557=0.318 \mathrm{GW}$ or 318 MW
d. A generating plant will have a nameplate rating, or rated capacity, which is the maximum power that it can sustain. The capacity factor is the average amount of time that a plant would operate at its nameplate rating. If the average capacity factor for the fleet of coal plants is 65\%, estimate the nameplate rating of the coal plants (assuming they were all the same capacity). (2)
- Capacity Factor = Actual Energy (or power) generated / Maximum possible energy (or power) generated
- The maximum power is the nameplate of the plant and maximum energy is the nameplate multiplied by $24 \mathrm{hrs} /$ day $\times 365$ days $=8760 \mathrm{hrs}$.
- In the problem above we assume they are operating $24 / 7$ with an power of 318 MW . However, since each of the plants is really only operating $65 \%$ of the time, the actual plant capacity must be bigger. Capacity Factor $=\frac{\text { ave power }}{\text { nameplate power }}$. So, rearranging, we have nameplate $=\frac{\text { ave power }}{\text { nameplate power }}=$ $\frac{318 M W}{0.65}=490 M W$ ( 2 sig digs).
e. The average US home used $11,300 \mathrm{kWh} /$ year in 2011 . How many coal plants (using the average size calculated above) would be needed to supply electricity to a city the size of Albuquerque? (2)
- We are assuming an average plant size of 490 MW operating with a 0.65 capacity factor. The wording of the question doesn't make it clear that you should use the capacity factor, so if you solved without it, that is fine, also. The solution to the problem will look like:
- US Census data, for 2009-2013 shows 222,491 households in ABQ.
- Number of power plants $=\frac{\text { total demand }}{\text { total supply per power plant }}=\frac{\frac{11.3 M W h}{y r} \times 222,491}{\frac{490 M W}{\text { powerplant }} \times 0.65 \times 8760 \mathrm{hrs} / y r}=0.90$ power plants. Therefore, about one 490MW power plant would be able to supply electricity to albuqurque.
f. A 1 kW solar array has a capacity factor of about $25 \%$. What would be the total installed capacity of solar needed to meet the demand of the same city? (2)
- A 1 kW solar array would generate operating at a capacity factor of 0.25 (why is the capacity factor so low for solar?) would generate $1 \mathrm{~kW} \times 0.25 \times 8760 \mathrm{hrs} / \mathrm{yr}=2190 \mathrm{kWh} / \mathrm{yr}$.
- There are a number of ways to think about setting up this problem. I will use the equation for capacity factor, but put it in terms of energy rather than power. Remember that energy is power times time, which is why we are multiplying the nameplate rating (always power) by the number of hours in a year. Capacity Factor $=\frac{\text { ave energy } / y r}{\text { nameplate powerx8760hrs. }}$. The nameplate power would be the installed capacity of solar (what we want), and the ave energy would be the amount of energy needed to meet Abq demand. Solving for nameplate power we have:
- Nameplate $(M W)=\frac{\text { ave energy } / y r}{\text { cap factor } x 8760 h r s / y r}=\frac{\frac{11.3 M W h}{y r} \times 222,491}{0.25 \times 8760 \mathrm{hrs} / \mathrm{yr}}=1.1 \mathrm{GW}$ of PV. To give you an idea of the scale of such an array, the largest array in the world right now is 0.5 GW , so this would be twice the size of the world's largest array.


## 3. Energy use patterns ( 10 pts )

- Visit the EIA and look up the amount of electricity that was consumed in the US in 2013. Look up on the UN the total population of the US. Calculate the average annual consumption per person in the US in 2013. (2)
- It turns out the data wasn't so easy to find on EIA or UN for 2013...Sorry. I found that the US had 316e6 people (us census, 2013), consumed $4,686,400,000$ MWh (US DOE). So the annual consumption was 14.8 MWh/yr per capita.
- Do you think this is a reasonable estimation for annual consumption in the typical home? How does it compare to the number you used for residential consumption in the calculation above? (2)
- This is probably not reasonable for residential consumption since the consumption data we used also included industry and commercial buildings. Looking at the Sankey diagram from lecture 2, we see that $38 \%$ of electricity is consumed in homes. The commercial buildings is reasonable to include if we want to know typical electricity consumption by people, not just in their homes. It was 1.3 times bigger than the consumption given in the earlier problem.
- Now look at the electricity bill where you live. Divide this number by the total number of people living in your house or apartment. How does this number compare? Given an explanation if it is lower or higher than the US average. (2)
- Many answers possible here....
- Now look up the most recent energy consumption of electricity in India in 2013 available on the International Energy Administration website (www.iea.org), as well as the current population. Calculate the average consumption per person. Repeat the same calculation for Malawi. Give an explanation why both of these numbers may be bigger or smaller than in the US. (4)
- I find for 2012 (they don't have 2013 on IEA) that India consumed 868,752 GWh total, and 190,870 GWh in residential consumption. The population in 2012 was 1237 million. Thus the per capita consumption was 0.7 $\mathrm{MWh} / \mathrm{yr}$ per person, or $0.1 \mathrm{MWh} / \mathrm{yr}$ per person if I use only residential consumption.
- I couldn't find data on Malawi on the IEA, but I did find consumption data on the EIA for 2012 as 1.93 million MWh. The population in 2012 was 15.91 million (World Bank). So the consumption per capita is $0.1 \mathrm{MWh} / \mathrm{yr}$ per person.
- Malawi has an electrification rate of only 7\%, while India has an electrification rate of $75 \%$ (IEA, 2013). However, given India's large population it has the largest unelectrified population in the world ( 306 million). This difference in electrification rates may account for why India's total consumption per capita is 7 times bigger. The US's total consumption ( $14.3 \mathrm{MWh} /$ person) is 20 x bigger than India's. Part of this is due to India's lower electrification rate, but also due to the higher rates of consumption in the US due to higher
income - owning bigger houses, AC, more appliances, etc. In order to compare apples to apples, we would only use residential consumption numbers, since differences in economy (industrial consumption) will skew the numbers.

